- **Goal:** Model and simulate car-following behaviors with heterogeneity and stochasticity.
- **Challenge:** Traditional models learn a *one-to-one mapping*

 $f_{\mathrm{CF}}:(s_t,\Delta v_t,v_t)\mapsto a_t$ 

(deterministic) X



but real drivers induce a *one-to-many mapping* with uncertainty

$$f_{\text{CF}}: (s_t, \Delta v_t, v_t) \mapsto \{a_t^{(1)}, a_t^{(2)}, \dots\}$$
 (stochastic)

**Solutions:** 

## **Explicit Uncertainty Modeling** (continuous variability)

 $a_tpprox f_{ ext{CF}}(m{x}_t;m{ heta})+\delta_t,$  (Zhang and Sun 2024, Zhang et al. 2024, Zhang et al. 2025a)

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Table 1: Modeling	of femnora	Lcorrelations	in my	previous w	zork
Table 1. Modelling	or tempora	Contenunons	111 111 y	previous w	OIK.

	<u> </u>	<u> </u>
Reference	$\int_{\mathrm{CF}}(\mathbf{x}_t; \mathbf{\theta})$	$\mid \delta_t$
Zhang and Sun (202	(4) IDM	Gaussian processes (GPs)
Zhang et al. (2024)	IDM	Autoregressive (AR) processes
Zhang et al. (2025a	) NN	nonstationary GPs

[Chengyuan Zhang and Lijun Sun. (2024). Bayesian calibration of the intelligent driver model. IEEE Transactions on Intelligent Transportation Systems.] [Chengyuan Zhang, Wenshuo Wang, and Lijun Sun. Calibrating car-following models via Bayesian dynamic regression. (ISTTT25 Special Issue)

Transportation Research Part C: Emerging Technologies 168 (2024): 104719.1

[Chengyuan Zhang, Zhengbing He, Cathy Wu, and Lijun Sun. (2025a). When Context Is Not Enough: Modeling Unexplained Variability in Car-Following Behavior. arXiv preprint arXiv:2507.07012 (under review).]

## Latent Variable Modeling (discrete variability)

$$a_t pprox f_{\mathrm{CF}}(m{x}_t; m{ heta}_{z_t}), \ z_t \sim egin{cases} \mathrm{Gaussian\ Mixture} & \textit{(i.i.d.)} \ \mathrm{(Chen\ et\ al.\ 2023,\ Zhang\ et\ al.\ 2024)} \ \mathrm{Markov\ Chain} & \textit{(temporal\ dependence)} \end{cases}$$

(Zhang et al. 2025b)

[Xiaoxu Chen, Chengyuan Zhang, Zhanhong Cheng, Yuang Hou, and Lijun Sun. A bayesian gaussian mixture model for probabilistic modeling of carfollowing behaviors. IEEE Transactions on Intelligent Transportation Systems 25, no. 6 (2023): 5880-5891.]

IChengyuan Zhang, Kehua Chen, Meixin Zhu, Hai Yang, and Liiun Sun, Learning car-following behaviors using bayesian matrix normal mixture regression. In 2024 IEEE Intelligent Vehicles Symposium (IV), pp. 608-613. IEEE, 2024.]

[Chengyuan Zhang, Cathy Wu, and Lijun Sun. (2025b). Markov Regime-Switching Intelligent Driver Model for Interpretable Car-Following Behavior. arXiv preprint arXiv:2506.14762 (2025b). (under review) .]









